From (11) we find for $x=f(y)$ the expression

$$x=\sqrt{\frac{a\sqrt{(2g)}}{n.\pi}}.y^{(2-n)/4n}\ldots.(12),$$

which for $n=1$, 2, 3, ... *i. e.*, for the first, square, and cube-root, becomes

$$x=\sqrt{\frac{a\sqrt{(2g)}}{\pi}}.y^{\frac{1}{4}}\ldots.(13),$$

$$x=\sqrt{\frac{a\sqrt{(2g)}}{2\pi}}\ldots.(14),$$

$$x=\sqrt{\frac{a\sqrt{(2g)}}{2\pi}}.y^{-\frac{1}{12}}\ldots.(15),$$

respectively. The shape of the vessel in case of square-roots is therefore cylindrical, and if the radius of the cylinder is made according to (14) *the time of emptying the cylindrical vessel will be equal to the square root of the original depth of water.*

The physical conditions of the problem make it clear that the first method is more accurate. The determination of the time in the second method is liable to be affected with an error of a higher order than those occuring in the statical extraction of a root.

In the near future the author shall publish a remarkably simple extension of the first method to the solution of an equation of the form

$$a_0+a_1x+a_2x^2+a_3x^3+\ldots.+a_nx^n=0.$$

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

133. Proposed by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

In Wentworth's Arithmetic he gives a formula $\frac{21}{40}(d^2-2d)$ for calculating the number of board feet in a log 10 feet long, when d is the diameter in inches. How is this rule derived?

Solution by P. H. PHILBRICK, C. E., Lake Charles, La.

The number of board feet in a log d inches in diameter and l inches in length is $n=\dfrac{\frac{1}{4}\pi d^2 l}{144}$.

If l is in feet, $n=\dfrac{\frac{1}{4}\pi d^2 l}{12}$.

Substituting $\frac{22}{7}$ for π and 10 for l we have, $n=\frac{5}{6}.\frac{11}{14}d^2$.

Allowing $\frac{1}{5}$ for saw cut, $n=\frac{4}{5}.\frac{5}{6}.\frac{11}{14}d^2=\frac{11}{21}d^2$.

Allowing $\frac{1}{2}$ inch for bark, $n=\frac{11}{21}(d-1)^2$.

For $d=22$, an average value, $(d-1)^2=\frac{441}{440}(d^2-2d)$.

$\therefore\ n=\frac{11}{21}.\frac{441}{440}(d^2-2d)=\frac{21}{40}(d^2-2d)$.

I would propose the formula $n=d^2$(or $n=(d-1)^2$) for a log 20 feet long, since it is as accurate and much more simple than Wentworth's.

The most accurate formula, however, must be based on the end diameters of the log.

Let d and D represent those diameters.

Board feet in total volume of log 20 feet long

$$=\tfrac{1}{4}\pi.\tfrac{5}{3}\times\frac{d^2+dD+D^2}{3}=\tfrac{5}{9}\times\tfrac{11}{14}(d^2+dD+D^2).$$

(See Philbrick's *Engineer's Manual*, table 23).

Since $(D-d)^2=D^2-2dD+d^2>0$, $D^2+dD+d^2>3dD$.

Hence volume$>\frac{5}{3}\times\frac{11}{14}dD$.

Allowing $\frac{1}{5}$ for saw cut, $n>\frac{4}{3}\times\frac{11}{14}dD=\frac{22}{21}dD$.

Allowing $\frac{1}{22}$ of the above for bark we still have $n>dD$, or, say, $n=dD$ ....(1).

It is easily shown that volume$>\frac{5}{3}\times\frac{11}{14}\left[\dfrac{d+D}{2}\right]^2$

and as before that $n=\left[\dfrac{d+D}{2}\right]^2$....(2).

The author's experience leads him to believe that the above formulas are quite accurate, but that logs will cut a little more into large timbers than the formulas give.

If thought to give too large a result, in extreme cases, we might suggest the formula, $n=d(D-2)$....(3), or $n=\left[\dfrac{d+D}{2}-1\right]^2$....(4).

At all events the forms suggested should be used.

ALGEBRA.

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111. Proposed by ARTEMAS MARTIN, A. M., Ph. D., LL.D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

Solve the equation $x(y+z)=a(x+y+z)$, $y(x+z)=b(x+y+z)$, $z(x+y)=c(x+y+z)$.